A SIMPLIFIED EXPLANATION, IN PHYSICAL TERMS, OF THE ACOUSTICAL CONSEQUENCES OF TONGUE AND LIP MOVEMENT IN VOWEL PRODUCTION

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Abstract: This paper presents an excerpt from a larger exposition, attempting to give non-technicians a basic insight in the relationship between articulation and acoustics of vowels and consonants.

By means of Newton's second law (force equals the product of mass and acceleration) and Boyle-Mariotte's law (at constant temperature the product of pressure and volume for a given quantity of air is constant) one can explain the fact that "When a part of a pipe is constricted its resonance frequency becomes low or high according as the constricted part is near the maximum point of the volume current ... or of the excess pressure ..." (Chiba & Kajiyama, 1958, p. 151). This is achieved mainly by considering the relative forces that act on a thin slice of air, oscillating back and forth through the open end of a quarter-wavelength resonator at its first resonance frequency: a decrease of the volume of the pipe near its closed end increases the forces that keep the slice in motion and thus raises its frequency, and vice versa. Inversely, diminishing the opening of the resonator decreases the forces that keep the slice in motion, and thus lowers its frequency, and vice versa.

1. Introduction

This paper does not pretend to be scientific and original in the ordinary sense of the words. I just try to explain to phoneticians without any special training in physics and mathematics, in a simpler fashion than do most of the articles and books on the subject, the often cited fact that "When a part of a pipe is constricted its resonance frequency becomes low or high according as the constricted part is near the maximum point of the volume current ... or of the excess pressure ..." (Chiba & Kajiyama, 1958, p. 151).

2. Initial simplifications

![Diagram of vocal tract and a simplified model of the vocal tract.](image)

**Figure 1**
The vocal tract and a simplified model of the vocal tract.

Suppose the vocal tract is a cylindrical pipe, 17.5cm long and with a diameter of 2.4cm, closed at one end, open at the other, see fig. 1. Let us say, further, that the walls of the pipe are perfectly hard and non-yielding (i.e. they do not absorb acoustic energy) and that there is no radiation of energy from the open end of the pipe to the exterior (i.e. there is no diffraction of sound from the lips. This is, of course, a monstrous absurdity, and in practice it would mean that we could not hear each other speak, but it is a convenient simplification and one which is not a serious obstacle to the qualitative considerations that follow.) Thus we are dealing with an ideal uniform quarter-wavelength resonator, with resonances at approximately 500, 1500, 2500, ... Hz (cf. p. 9).

3. The vibratory pattern in the uniform quarter-wavelength pipe at its first resonance frequency

As point of departure, let us consider the uniform pipe of fig. 1 (i.e. the neutral vowel). Let us look at the column of air, after it has been made to oscillate at its first resonance, and let us suppose that this oscillation continues with undiminished amplitude so long as we are interested in studying it. (In practice this is impossible without a constant supply of energy; this is of no consequence for the present treatment.)
The distribution of pressure variation (a) and volume velocity (b) in a uniform quarter-wavelength pipe at its first resonance.

Figure 2

We know that at the open end of the pipe (the lips) the variation of volume velocity\(^1\) is maximum, i.e. the air particles perform oscillations in and out through the opening with maximum elongation. At the closed end (glottis) the pressure variation\(^2\) is maximum. We also know that there is pressure variation and volume velocity, respectively, all along the pipe but that the pressure variation decreases from closed to open end (where it is zero) and that the velocity decreases from open to closed end (where it is zero), see fig. 2. (These facts can also be illu- cated in an intuitively comprehensible fashion, but not without exceeding the limits of this paper.)

However, as long as we are dealing with only the first resonance, we can conceive of the column of air as if its movement, i.e. the volume velocity, were concentrated at the open end and as if the pressure variation were concentrated at the closed end of the pipe. (Thus we are dealing with a system of concentrated constants, with one degree of liberty, i.e. it can oscillate at one, and only one, frequency.) In this case the acoustic system can be likened to a mechanical system, composed of a mass and a spring, attached to a hard wall, sliding on a perfectly smooth surface, which means that no friction occurs between the surface and the mass, when it oscillates, see fig. 3.

All movement presupposes a force: if the mass is displaced to the left (3b) the compression of the spring exerts a force to the right, and when we let go of the mass this force will set the

1) "variation of volume velocity" is occasionally abbreviated "volume velocity", or just "velocity" in the following.

2) "pressure variation" is occasionnally abbreviated "pressure" in the following.
mass in motion, towards its rest position. The mass will pass the rest position (3c), because every body that has a mass has inertia as well, which means that a movement will continue some time after the force which initiated it has ceased to operate. Thus, the spring becomes more and more stretched and exerts a growing force to the left which will eventually stop the motion of the mass (3d) and a movement to the left begins, towards the rest position. Because of its inertia, the mass will once more pass its position of equilibrium (3e), the spring is compressed anew and exerts a growing force to the right until the mass is stopped (3f) and a movement to the right commences, and so on and so forth. If no energy is lost anywhere, the mass will oscillate eternally with undiminished elongation. Its frequency depends on the tension of the spring and the size of the mass: the greater the tension, and the smaller the mass, the higher its frequency of oscillation, and vice versa. The elongation of the mass depends only on the initial displacement which sets the system in motion.

In the same fashion we can consider the behaviour of a thin slice of air, S, at the open end of the pipe, see fig. 4 (the movements and thickness of this slice are greatly exaggerated in the figures). What keeps this slice of air in motion is the combined action of (1) the pressure variations that arise in the
Vibratory pattern of a thin slice of air at the open end of the uniform quarter-wavelength pipe.

pipe due to the motion of S and (2) S's inertia. When S commences a movement to the right (4a) it is because the pressure in the pipe is greater than the atmospheric pressure outside the pipe, and when S passes its rest position (4b) it is because S has a certain (however small) mass and therefore inertia. Thereby the pressure within the pipe decreases, and the atmospheric pressure constitutes a (relative to the pressure in the pipe) growing force to the left, which eventually stops S and sets it in motion back towards the position of equilibrium (4c). This oscillation, too, will continue eternally, with undiminished amplitude, if no energy is lost anywhere from the system.

There is a close tie between the forces that act on S, S's mass, and S's motion, which is given by Newton's second law:

\[ [1.1] \quad F = m \cdot G \quad \text{(force equals the product of mass and acceleration)} \]

a) the force, in our case, is the product of pressure and the area of the surface to which the pressure applies:

\[ [1.2] \quad F = P \cdot A \]

This area is constant (see fig. 4). Only the pressure in the pipe varies.

b) S is simultaneously influenced by two antagonistic forces, one due to the pressure in the pipe and one due to the atmospheric pressure outside. The resultant force is due to the difference between these two pressures.
c) S's mass is constant.

Thus, one can paraphrase Newton's second law:

\[ 1.3 \quad P \cdot A = m \cdot G \quad \text{i.e.} \]

\[ 1.4 \quad (P_0 + \Delta P) \cdot A = m \cdot G \quad \text{thus} \quad \Delta P = \frac{m \cdot G}{A} \quad \text{or} \]

\[ 1.5 \quad \Delta P = k \cdot G \]

where \( P_0 \) is atmospheric pressure, \( \Delta P \) is the pressure increment (or decrement) in the pipe, \( k \) is a constant, equal to the mass of \( S \) divided by \( S \)'s surface area, and \( G \) is \( S \)'s acceleration, which can be taken as an indication of \( S \)'s mean velocity. Thus \( S \)'s velocity varies according to the difference in pressure within and outside the pipe. This difference is positive and negative, intermittently, and \( S \) thus moves from left to right and back again through the opening of the pipe.

4. Non-uniform pipes with constant opening

It can be shown that

a) \( S \)'s elongation depends only on the magnitude of the initial force.

b) \( S \)'s elongation and frequency are independent of each other.

c) \( S \)'s frequency depends only on the relative changes of pressure that are induced in the pipe due to \( S \)'s motion, which, in their turn, are determined by the total volume of the uniform pipe.

All this is a consequence of Newton's second law and of another law which states that, at constant temperature, the product of pressure and volume for a given quantity of air is constant (Boyle-Mariotte's law):

\[ 2.1 \quad P \cdot V = k \]

In our case it means that when the volume of the column of air is increased by \( S \)'s movement out of the pipe, the pressure in the pipe decreases, and vice versa. The most important fact to note is that as long as the volume increments and decrements are small compared to the total volume of the pipe, the pressure and volume variations are proportional to one another:
Let $P_0$ and $V_0$ be pressure and volume, respectively, in the rest condition:

$$[2.2] \quad P_0 \cdot V_0 = k$$

We decrease the volume by $\Delta V$ and get a pressure increase of $\Delta P_1$:

$$[2.3] \quad (P_0 + \Delta P_1)(V_0 - \Delta V) = k \quad \text{i.e.}$$

$$[2.4] \quad \Delta P_1(V_0 - \Delta V) = k - P_0(V_0 - \Delta V) = P_0 \cdot V_0 - P_0(V_0 - \Delta V) = P_0 \cdot \Delta V$$

We decrease the volume by $2\Delta V$ and get a pressure increase of $\Delta P_2$:

$$[2.5] \quad (P_0 + \Delta P_2)(V_0 - 2\Delta V) = k \quad \text{i.e.}$$

$$[2.6] \quad \Delta P_2(V_0 - 2\Delta V) = k - P_0(V_0 - 2\Delta V) = P_0 \cdot V_0 - P_0(V_0 - 2\Delta V) = 2P_0 \cdot \Delta V$$

thus

$$[2.7] \quad \Delta P_2 = \frac{2 \cdot P_0 \cdot \Delta V}{(V_0 - 2\Delta V)} = \frac{2 \cdot \Delta P_1(V_0 - \Delta V)}{(V_0 - 2\Delta V)} \approx 2 \cdot \Delta P_1 \quad \text{if } \Delta V \ll V_0$$

It follows that if a volume decrement of $\Delta V \text{cm}^3$ causes a pressure increment of $\Delta P \mu \text{bar}$, a volume decrement of $2\Delta V \text{cm}^3$ will render a pressure increment of $2\Delta P \mu \text{bar}$. (In practice the volume changes are very small indeed, since the elongation of the air particles is of an order of magnitude of a few millionths of a millimeter.)

Re (a) and (b) (p. 6)

Let us say that the initial force which sets $S$ going is a displacement to the left by $X \text{cm}$ (see fig. 5a). This produces a pressure of $(P_0 + \Delta P) \text{bar}$ in the pipe. When we let go of $S$ it starts moving to the right, and we know of its acceleration (and thus its mean velocity) that it is proportional to the difference between the pressures within and outside of the pipe.

$$[3.1] \quad (P_0 + \Delta P) - P_0 = k \cdot G_1 \quad \text{i.e.} \quad G_1 = \frac{\Delta P}{k}$$

If, instead, we commence by giving $S$ a displacement of $2X \text{cm}$ to the left (see fig. 5b), the pressure within the pipe will be $(P_0 + 2\Delta P) \text{bar}$. We get an acceleration as follows:
The relationship between initial elongation of $S$ and pressure increment in the uniform pipe.

\[ G_2 = \frac{(P_0 + 2\Delta P) - P_0}{k} = \frac{2\Delta P}{k} \]

thus \[ G_2 = 2G_1 \]

The mean velocity of $S$ in the second case is twice that of $S$ in the first case, but the elongation is also twice that of the first case, and thus the frequency of oscillation is identical in the two cases, and is independent of the elongation.

The relationship between pressure increment and elongation of $S$ in two uniform pipes with volumes $V_{cm}^3$ (a) and $\frac{1}{2}V_{cm}^3$ (b).

Re (b) and (c)

If one displaces $S_1$ and $S_2$ of fig. 6 by $X_{cm}$ to the left, in pipes having volumes of $V_{cm}^3$ and $\frac{1}{2}V_{cm}^3$, the relative volume decrement in the lower pipe is twice that of the upper pipe, and thus the pressure increment in the lower pipe is twice that of the upper pipe. The force which acts on $S_2$ is thus twice as large as the one that acts on $S_1$. The mean velocity of $S_2$ is therefore twice that of $S_1$, and since their elongations are identical, the frequency of oscillation of $S_2$ must be twice as high as that of $S_1$. (This is in complete accord with what we obtain from the formula for resonance frequencies in uniform quarter-wavelength
pipes:

$$f_n = \frac{c}{4 \cdot L} (2n-1)$$

where $c$ is the speed of sound in air, $L$ is the length of the pipe, $n$ is the number of the resonance. If $c=35000$ cm/sec, $L_1=17.5$ cm (a), and $L_2=8.75$ cm (b) we get:

(a): $f_1 = \frac{35000}{70} = 500$ Hz  
(b): $f_1 = \frac{35000}{35} = 1000$ Hz

The model of oscillation described above is extremely simplified, because velocity and pressure are not concentrated at the open and closed ends, respectively, of the pipe, see fig. 2. In practice this means that a volume change will have the greatest influence on the first resonance if it is located near the closed end of the pipe, where pressure variation is at its maximum.

Figure 7  Models of two vowels [a] and [i].

We may conclude that the frequency of the first resonance of the pipe in fig. 7 above, which is a model of the vowel [a], must be higher than that of the uniform pipe ([e]) and that, inversely, the first resonance of the pipe in fig. 7 below, which is a model of the vowel [i], is lower than that of [e], which is confirmed by empirical facts. (See also the summary.)

If we wish to consider the effect of volume changes upon the second, third, etc., resonances, we can no longer compare the system with a single slice of air (one mass) and one volume with pressure variation (one spring). The pressure distribution along the pipe at its second resonance frequency is depicted in fig. 8. If the column of air oscillates only at its second resonance the system behaves exactly as if it were combined of three pipes, each 1/3 Lcm long. The two imaginary pipes to the left in fig.
The pressure distribution in a uniform pipe at its second resonance frequency.

8b are joined with the open ends against each other, and the pipe to the right is joined, closed ends together, to the one in the middle. Now we can reason about the system in the same way as for the first resonance, only there are two places where a change of volume will have an appreciable effect on the (second) resonance, namely at the closed end and at a distance of 2/3 L cm from the closed end. At the third resonance there will be three places, at the fourth four places, etc., where volume changes will affect the resonance frequency appreciably. Each resonance has a pressure maximum at the closed end of the pipe, and thus all resonance frequencies rise or lower as a consequence of a decrease or increase of the volume near the closed end (but not always to the same degree, see the summary).

5. Uniform pipes with varying degrees of opening

The relationship between the elongation of two slices of air with areas of A cm$^2$ (a) and $\frac{1}{2}$A cm$^2$ (b) and pressure increment in the uniform pipe.

Let us now consider what happens if we decrease or increase the volume at the open end of the pipe. The volume change in itself is of no consequence, since there is no pressure variation
at the open end of the pipe, but the vertical closening or opening of the pipe is essential, and it is that only which is depicted in fig. 9. We compare two pipes of identical diameters and lengths. The upper pipe, $R_1$, is fully open, i.e. the area of opening is $Acm^2$. The lower pipe, $R_2$, has a circular opening of $\frac{1}{2}Acm^2$. The two slices, $S_1$ and $S_2$, are equally thick, $Bcm$. We employ the laws of Newton and of Boyle–Mariotte: $F = m \cdot G$ and $P \cdot V = k$.

The masses of $S_1$ and $S_2$ are known if we know the volume of the slices and the density of the air, $\rho$:

\begin{equation}
[4.1] \quad m_1 = A \cdot B \cdot \rho
\end{equation}
\begin{equation}
[4.2] \quad m_2 = \frac{1}{2}A \cdot B \cdot \rho
\end{equation}

For a given pressure increment, $\Delta P$, in $R_1$ and $R_2$, we get forces, $F_1$ and $F_2$, that act on $S_1$ and $S_2$ as follows:

\begin{equation}
[5.1] \quad F_1 = \Delta P \cdot A
\end{equation}
\begin{equation}
[5.2] \quad F_2 = \Delta P \cdot \frac{1}{2}A
\end{equation}

But force also equals the product of mass and acceleration, thus:

\begin{equation}
[6.1] \quad F_1 = \Delta P \cdot A = m_1 \cdot G_1 \quad i.e. \quad G_1 = \frac{\Delta P \cdot A}{m_1} = \frac{\Delta P \cdot A}{A \cdot B \cdot \rho} = \frac{\Delta P}{B \cdot \rho}
\end{equation}
\begin{equation}
[6.2] \quad F_2 = \Delta P \cdot \frac{1}{2}A = m_2 \cdot G_2 \quad i.e. \quad G_2 = \frac{\Delta P \cdot \frac{1}{2}A}{m_2} = \frac{\Delta P \cdot \frac{1}{2}A}{\frac{1}{2}A \cdot B \cdot \rho} = \frac{\Delta P}{B \cdot \rho}
\end{equation}

thus \begin{equation}
[6.3] \quad G_1 = G_2
\end{equation}

The two slices of air will have the same acceleration (mean velocity). BUT they do not have the same elongation. In order to induce in $R_1$ and $R_2$ the same volume decrement, and thus the same pressure increment, $S_2$ will have to be displaced twice as far into the pipe as $S_1$, because the surface area and volume of $S_2$ are only half those of $S_1$. If the two slices have the same mean velocity, but the distance covered by $S_2$ is twice that covered by $S_1$, $S_2$'s period will be twice that of $S_1$, and consequently the frequency of oscillation of $S_2$ will be half that of $S_1$.

If, instead, we commence by giving $S_1$ and $S_2$ the same displacement, as in fig. 10, we know that the volume decrement in $R_1$ is twice that of $R_2$. The pressure increment in $R_1$ is thus twice that of $R_2$, e.g. $2\Delta P$ as against $\Delta P$. These values are sub-
The relationship between initial elongation of $S_1$ and $S_2$ and pressure increment in the uniform pipe.

\[ G_1 = \frac{2 \Delta P}{2 \times \text{cm}} \quad \text{(7.1)} \]
\[ G_2 = \frac{\Delta P}{\text{cm}} \quad \text{(7.2)} \]

i.e. \[ G_1 = 2G_2 \quad \text{(7.3)} \]

The mean velocity of $S_1$ will be twice as large as that of $S_2$, and since the elongations are identical, the frequency of oscillation of $S_1$ will be twice as high as that of $S_2$.

Since the volume velocity is not concentrated at the open end, but is distributed all along the pipe (see fig. 2) we may conclude that an occlusion will have a greater effect on the first resonance frequency near the open end of the pipe, where velocity is at its maximum.

![Velocity distribution in a uniform pipe at its second resonance.](image11)

If we consider the second, third, etc., resonances we must again look at the velocity distribution all along the pipe. At the second resonance (see fig. 11) there are two places where the variation of volume velocity is maximum, namely $1/3 \text{ Lcm}$ from the closed end, and at the open end. At the third resonance there will be three places, at the fourth four places, etc., where an occlusion will have an appreciable effect on the resonance. Each resonance has a velocity maximum at the open end and thus all resonances are lowered by an occlusion at the open end (but not
always to the same degree, see the summary below).

6. Summary

A volume increase in the pipe produces a lowering of a resonance (and vice versa), the more so the nearer it is located to a pressure maximum for that resonance, and an occlusion produces a lowering of a resonance (and vice versa), the more so the nearer it is located to a velocity maximum for that resonance, and, all things being equal, the greater the change in volume or opening/closing, the greater the change in frequency. However, in practice we cannot separate these two types of changes within the vocal tract. Because of the limitations imposed by the articulatory organs, variations in the cross-sectional area within the vocal tract are simultaneously volume changes and occlusions/openings. Therefore the general formulation "When a part of a pipe is constricted its resonance frequency becomes low or high according as the constricted part is near the maximum point of the volume current or of the excess pressure."

It follows that if the constriction or expansion is situated exactly between a pressure and a velocity maximum, it will have no effect. Further: the vocal tract is an integrated system whose configuration is determined by the position of the tongue and lips. The tongue cannot simultaneously perform an extended constriction in the pharynx and at the hard palate, on the contrary, a pharyngeal constriction produces a relative expansion near the hard palate, and vice versa, see fig. 7. The cumulative effect is a "double" raising ([ɑ]) or lowering ([ɨ]) of the first resonance frequency.

We have considered the acoustic consequences of changes in the cross-section of the vocal tract for each resonance separately. In practice a vowel is, of course, always composed of several resonances, that conjointly form one complex oscillation. This is of no consequence for our considerations - one can treat this complex oscillation as a superposition of sinusoidal oscillations and consider the effect of a change in the vocal tract for each component separately.

What is more important is the fact that as soon as one does not as point of departure take the uniform pipe, but a pipe al-
ready deformed (like the one in fig. 7 below, [i]) one cannot quantify the changes in frequency in as simple a fashion as for the uniform pipe. This is due to the fact that the distribution of pressure and velocity is no longer sinusoidal (as it is in figs. 2, 8, 11). One example will suffice: for the model of [i] the velocity at the second resonance is very nearly zero in the front third of the pipe near the opening, and an occlusion at the opening (rounding of the lips) will thus have very little effect on the second resonance. But since the velocity at the third resonance is maximum (greater, in fact, than for the uniform pipe) it will decrease radically due to an occlusion (rounding) at the opening. (For diagrams of the distribution of pressure and velocity for several vowels and resonance frequencies, see the works cited in the references.) This is why one can say, not wholly unjustified, that certain resonances are, in certain cases, more dependent on changes in one part of the vocal tract than in another, and this is true especially of the narrow vowels.

7. Conclusion

In real life, i.e. speech, the situation is far more complicated than this demonstration would lead one to believe. The walls of the vocal tract are not hard, and there is a considerable radiation of energy to the exterior. Apart from the loss of energy, this radiation causes a tuning of the resonances, which is not of the same magnitude for high and low frequencies, and it is, further, heavily dependent on the degree of opening at the lips. The voice source, i.e. the pulse train from the glottis, constitutes another complicating element, among other things by the coupling it allows between the sub- and supraglottal cavities. Apart from all that, the mathematics and physics employed above do not suffice: in order to quantify the consequences of tongue and lip movement, one must solve higher order differential equations.
References

